Slotted Line :

* It's used to determine unknown load Z_L for mismatched load $Z_L \neq Z_0$

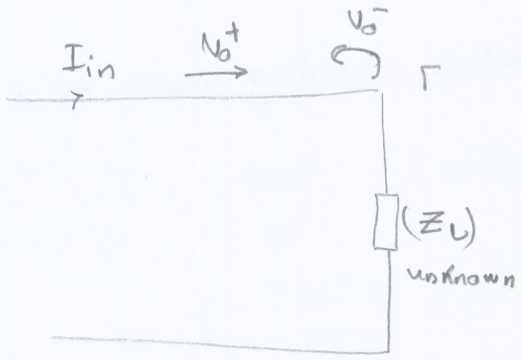
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma Z_L + \Gamma Z_0 = Z_L - Z_0$$

$$Z_L(\Gamma - 1) = -(\Gamma + 1) Z_0$$

$$Z_L = \frac{1 + \Gamma}{1 - \Gamma} Z_0 \quad \#$$

$$\Gamma = |\Gamma| e^{j\theta}$$



* From the wave eqn of T.L :

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{j\beta z}$$

* For lossless T.L $\quad \gamma = \alpha + j\beta = j\beta$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \qquad V_0^- = \Gamma V_0^+$$

$$V(z) = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}$$

$$= V_0^+ e^{-j\beta z} [1 + \Gamma e^{2j\beta z}]$$

$$|V(z)| = V_0^+ |1 + \Gamma e^{2j\beta z}| \qquad \text{but} \quad \Gamma = |\Gamma| e^{j\theta}$$

$$|V(z)| = V_0^+ |1 + |\Gamma| e^{j(\theta + 2\beta z)}| \quad \#\#$$

$$|V(z)| = V_0^+ \left| 1 + |\Gamma| e^{j(\theta + 2\beta z)} \right| \quad \text{---} \quad (1)$$

$$V_{max} = V_0^+ (1 + |\Gamma|)$$

$$V_{min} = V_0^+ (1 - |\Gamma|)$$

Voltage standing wave ratio :

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{V_0^+ (1 + |\Gamma|)}{V_0^+ (1 - |\Gamma|)} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1} \quad **$$

From (1) : The magnitude of the voltage at any distance l from the load is given by :

* let $z = -l$

$$|V(l)| = V_0^+ \left| 1 + |\Gamma| e^{j(\theta - 2\beta l)} \right|$$

* the first min voltage occur of distance l_{min} from the load :

$$V_{min} = |V(l_{min})| = V_0^+ \left| 1 + |\Gamma| e^{j(\theta - 2\beta l_{min})} \right|$$

$$e^{j(\theta - 2\beta l_{min})} = \cos(\theta - 2\beta l_{min}) + j \sin(\theta - 2\beta l_{min}) = -1$$

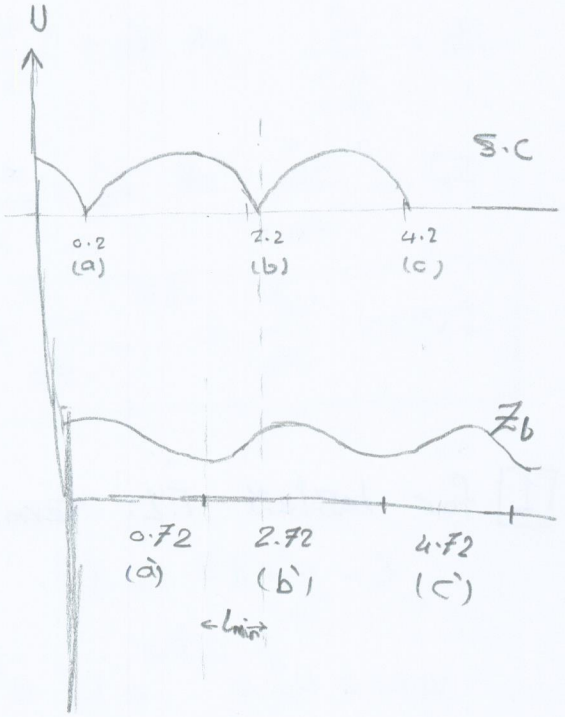
$$\therefore \cos(\theta - 2\beta l_{min}) = -1$$

at $\theta - 2\beta l_{min} = \pi \implies \theta = \pi + 2\beta l_{min}$

Ex : Coaxial slotted line with $Z_0 = 50\ \Omega$. detemine Z_L with zero voltage (short circuit) at 0.2cm, 2.2 Cm, 4.2 cm

Sol

$$L_{min} = B - A' = 2.2 - 0.72 = 1.48 \text{ cm}$$



$$\frac{\lambda}{2} = B - A = 2.2 - 0.2 = 2 \text{ Cm}$$

$$\therefore \lambda = 4 \text{ Cm}$$

$$\frac{L_{min}}{\lambda} = \frac{1.48}{4} \Rightarrow L_{min} = 0.37\ \lambda$$

$$VSWR = 1.5 \Rightarrow |\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$|\Gamma| = 0.2$$

$$\theta = \pi - 2\beta L_{min}$$

$$= \pi - 2\,\frac{2\pi}{\lambda} \cdot 0.37\lambda = 7.79 \text{ rad} = 86.4^\circ$$

$$\Gamma = |\Gamma| e^{j\theta} = 0.2\, e^{j86.4}$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\, \frac{1 + 0.2 e^{j86.4}}{1 - 0.2 e^{j86.4}}$$

$$= 47.3 + j\,19.7\ \Omega$$

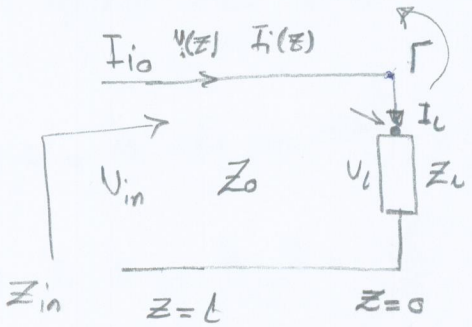
* For a T.L. Terminated by a Load Z_L

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I(z) = I_o^+ e^{-\gamma z} - I_o^- e^{\gamma z}$$

$$Z_o = \frac{V_o^+}{I_o^+} \quad \text{OR} \quad Z_o = \frac{-V_o^-}{I_o^-}$$

$$I_o^+ = \frac{V_o^+}{Z_o} \quad \text{OR} \quad I_o^- = \frac{-V_o^-}{Z_o}$$



$$\boxed{I(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}} \quad \text{——} \quad (1)$$

1 for Lossless T.L. terminated by load Z_L :

$$\gamma = \cancel{\alpha}^{0} + j\beta = j\beta$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$$

* At load : $z = 0$

$$\therefore V(0) = V_o^+ + V_o^- = V(L)$$

$$I(0) = \frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o} = I(L)$$

$$Z_l = \frac{V_L}{I_L} = \frac{V_o^+ + V_o^-}{\frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o}} = Z_o \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} \qquad \text{but} \quad \Gamma = \frac{V_o^-}{V_o^+}$$

$$= Z_o \frac{1 + \frac{V_o^-}{V_o^+}}{1 - \frac{V_o^-}{V_o^+}} = Z_o \frac{1+\Gamma}{1-\Gamma} \Rightarrow Z_L - \Gamma Z_L = Z_o + \Gamma Z_o$$

$$\Gamma(-Z_L - Z_o) = -Z_L + Z_o \longrightarrow \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V(-l)}{I(-l)} = \frac{V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}}{\frac{V_o^+}{Z_o} e^{j\beta l} - \frac{V_o^-}{Z_o} e^{-j\beta l}} = Z_o \frac{V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l} - V_o^- e^{-j\beta l}}$$